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Regularities of unsteady radiative–conductive heat transfer in evaporating semitransparent liquid droplets

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Abstract

The numerical investigation method of unsteady transfer processes in evaporating droplets in radiating media is introduced, evaluating the dependence of optical spectral properties of material upon temperature. The distribution of temperature and heat fluxes regularities in heating and simultaneously evaporating water droplets has been investigated. It is shown that as a cause of interaction of radiation and conduction processes, the profile of the temperature field inside the droplet is distorted, and the magnitude and direction of heat conductivity flux vector changes. According to the maximum place in the instant temperature field of the droplet, it is suggested to distinguish three periods of state change for an evaporating droplet: initial, transient and final. The results of the unsteady radiative–conductive heat transfer are generalized by using similarity theory methods.

1. Introduction

The analysis of heat and mass transfer in liquid droplets, known as the 'droplet problem', is important for the knowledge of thermal technological processes in dispersed system regularities and natural phenomena. Investigations of the 'droplet problem' are important not only for the optimization of traditional liquid fuel burning and high temperature gas cooling processes [1–6], but also for various non-traditional problems like dispersed working body (liquid metal) cooling possibilities in space shuttles [7], regularities of meteoritic descent in the atmosphere [8], emergency cooling of nuclear reactors' active zone with liquid (water) jets [9].

Thorough scientific work reviews about 'droplets' showing the research stages in different periods of the 20th century are presented in [2,3,10–13]. Heat and mass transfer in pure liquid droplets, evaporating in low temperature media was discussed in earlier research [10]. The processes of energy and mass transfer were thoroughly discussed in later studies. Several important factors were evaluated, i.e. gas flow, process unsteadiness, the dependence of radiation and physical properties on temperature, also the influence of the Stefan flow on energy and mass transfer. Transfer processes inside and outside a droplet are closely related. The whole complex of these transfers influences the interphase contact surface temperature, the magnitude of which can be defined by the equation of energy balance on the droplet surface:

$$\sum \vec{q}(R^{+}) + \sum \vec{q}(R^{-}) = 0, \tag{1}$$

which requires that all incoming and outgoing droplet surface energy fluxes must be equal. From a mathematical point of view, condition (1) is a system of integral–differential equations. Their solution is quite complicated, therefore the 'droplet' problem is solved using certain premises. Under the statement that

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Nomenclature

a	thermal diffusivity
B	Spalding transfer number
c_p	mass specific heat
D	mass diffusivity
h	enthalpy per unit mass
I	control time index
I_ω	intensity of radiation
$I_{\omega 0}$	spectral index of radiation of a black body
k_e	effective conductivity parameter
k_ω	spectral index of absorption
L	latent heat of evaporation
m	mass flux density
n	number of the term in infinite sum
n_ω	spectral index of refraction
N	number of terms in infinite sum
Nu	Nusselt number
p	pressure
Pe	Peclet number
Pr	Prandtl number
q	heat flux density
R	radius of a droplet
r	coordinate of a sphere
r_ω	spectral index of light reflection
Re	Reynolds number
R_*	universal gas constant
t	time
T	temperature

Greek symbols

ω	wave number
χ	dimensionless interaction parameter
Δw	slip velocity of a droplet in gas
β, γ	angles, estimating the peculiarities of spherical geometry when calculating radiation
η	dimensionless coordinate
φ	azimuthal angle
λ	thermal conductivity; wavelength
μ	molecular mass
θ	angle between the opposite direction of the normal to the surface element and the incident beam
ρ	density
τ	optical thickness

Subscripts

b	source of radiation
f	evaporation
g	gas
i	time index in a digital scheme
I	index of control time
J	index of sphere cross-section
c	convective
l	conductive
m	mass average
r	radiation
s	surface of an interphase contact
v	vapor
vg	gas–vapor mixture
ω	spectral
Σ	total
0	initial state
∞	far from a droplet

Superscripts

k	number of iterations
$'$	variable
0	interaction free
'+'	external side of a surface
'−'	internal side of a surface

droplet evaporation is in equilibrium, the unsteadiness of transfer processes can thus be neglected [10,14–17]. In such a case all the energy, which a droplet gets from outside, is used for the phase changes. This premise neglects the importance of the internal problem, i.e. a droplet is said to be isothermal. The problem of evaporating droplet outer convective heating has been successively solved using the methods of similarity theory, taking into account the interaction of convection and mass transfer. In that case the intensity of the evaporating droplet convective heating is calculated using well-known criterion equations which define the convective heat transfer of a solid sphere. These equations are modified introducing the corrections evaluating the influence of mass transfer on convection [14–20].

When the temperature of the liquid dispersed into gas is smaller than the equilibrium evaporation temperature corresponding to the boundary heat and mass transfer conditions, a droplet is being heated up intensively, therefore transfer processes are distinctly unsteady, hence it is impossible to neglect the inner 'droplet' problem. When radiative heat transfer is not taken into account, the inner 'droplet' problem can be solved easily using 'infinitive conductivity' and 'rapid-mixing limit' models [12]. According to them, the transfer processes inside a droplet are so intensive that in transient heating the temperature within the droplet is spatially uniform. When thermal conductivity of the liquid is finite the temperature field is defined by the 'conduction-limit' mathematical model [3]. The influence of liquid circulation inside the droplet on heat transfer is taken into account in the 'effective conductivity' model [19]. But these droplet internal heat transfer mathematical models cannot be used when it is necessary to calculate the combined energy transfer in evaporating semitransparent liquid droplets in radiating media. In this case it is necessary to use additional mathematical models that describe energy transfer by radiation in a droplet. Simply the energy supplied to a droplet by radiation from outside can be calculated according to integral [21,22] and monochromatic [23,24] radiative properties. Various spectral models of heat transfer by radiation enable us to evaluate the peculiarities of semitransparent liquid optical characteristics [15,25–27], the influence on the transfer processes of the dependence of these peculiarities on liquid temperature [28] and the radiation absorption in a droplet. Usually the interaction between different transfer mechanisms is not taken into account and only the energy brought to a droplet by radiation is calculated. The necessity of the evaluation of the above-mentioned interaction has been proved [26,29]. The calculation of combined energy transfer becomes simpler when convection and conduction components of the total energy flux are calculated by neglecting the influence of radiation and the interaction of different transfer mechanisms is evaluated by similarity methods [30,31]. 'Single' droplet models allow simulating the dispersed systems. State transformation of the dispersed systems is calculated according to the total energy use for heating of condensed media and phase

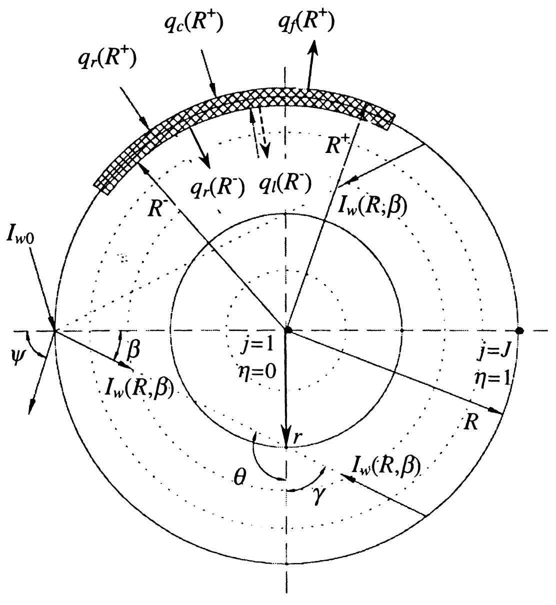

Fig. 1. Energy balance on the surface of an evaporating droplet.

changes. Thus, the equations describing 'droplet' internal and external problems have to be joined to the equations describing the state of a gaseous phase [32].

This work is devoted to the study of unsteady radiative–conductive heat transfer regularities in evaporating semitransparent liquid droplets.

2. Problem formulation

The energy balance (Eq. (1)) for dispersed semitransparent liquid droplet in a radiating media can be expanded (Fig. 1):

$$q_r(R^+) + q_c(R^+) - q_f(R^+) - q_\Sigma(R^-) = 0. \quad (2)$$

The interaction of radiation and convection processes is insignificant [33] in traditional thermal technologies; therefore, it is possible to calculate radiative and convective components of a droplet outside combined heating separately. The mass flux from an evaporating droplet dynamically influences the temperature and velocity fields of a gas flow passing around the droplet. The mass flux from the evaporating droplet also changes gas composition; therefore, it is necessary to evaluate mass transfer influence on the outer convective heating. The outer convective heating intensity of an evaporating spherically symmetric droplet can be calculated according to well-known equations which define the convective heat transfer of a non-evaporating sphere. These equations should be multiplied by a certain Spalding heat transfer number B function

$$Nu_v = Nu \cdot f(B). \quad (3)$$

Here Spalding heat transfer number B equals:

$$B = \frac{c_{pg}(T_g - T_s)}{L + (q/m)} \quad (4)$$

Generally in Eq. (4) q means the heat flux in the inner side of a droplet surface [34]. The magnitude of q is proportional to the temperature gradient in the droplet. q is assumed to be positive when its vector is directed towards the droplet center. Vapor mass flux m is assumed to be positive when a droplet is evaporating. Various expressions for the calculation of function $f(B)$ and convective heating intensity Nu for a non-evaporating sphere can be found in the literature. The investigation results of heat transfer in evaporating droplets are in good correlation when the intensity of convective heat transfer is calculated by [16,17]:

$$Nu_v = (2 + 0.57 Re^{1/2}\, Pr^{1/3})(1 + B)^{-0.7}. \quad (5)$$

In the case of equilibrium droplet evaporation all outer heating energy is used for droplet evaporation therefore, the liquid–vapor mass flux depends on the total heat flux supplied to the droplet:

$$m = \frac{q_\Sigma(R^+)}{L}, \tag{6}$$

and in Eq. (4) q corresponds to the radiation energy that has been absorbed in a droplet and brought to its surface, hence $q \equiv -q_r(R^-)$. Since the spectral radiation absorption coefficients are finite, radiation is not absorbed in a droplet surface

$$q_r(R^-) = q_r(R^+). \tag{7}$$

Taking into account expressions (4)–(7), heat flux by external convection in the case of droplet equilibrium evaporation can be calculated by the equation:

$$q_c(R^+) = \frac{\lambda_{vg}(T_g - T_s)}{2R}(2 + 0.57Re^{1/2}\, Pr^{1/3}) \times \left\{1 + \frac{c_{pg}(T_g - T_s)}{L}\left[1 + \frac{q_r(R^+)}{q_c(R^+)}\right]\right\}^{-0.7}. \tag{8}$$

During the unsteady droplet evaporation the liquid–vapor mass flux depends on the flux difference between the heat supplied to the droplet and the heat taken inside the droplet:

$$m = \frac{q_\Sigma(R^+) - q_\Sigma(R^-)}{L}, \tag{9}$$

and in Eq. (4) q corresponds to the difference between the total heat flux and radiation flux in a droplet:

$$q \equiv q_\Sigma(R^-) - q_r(R^-). \tag{10}$$

In order to calculate m and q in Eq. (4), it is necessary to know the total heat fluxes $q_\Sigma(R^+, t)$ and $q_\Sigma(R^-, t)$ and to evaluate the direction of temperature gradient vector in a droplet. Therefore, it is necessary to calculate the temperature field in a droplet.

When a combined energy transfer takes place in a semitransparent droplet, the total heat flux can be calculated by summing up the components of radiative and conductive flux:

$$q_\Sigma(R^-) = q_r(R^-) + k_e q_l(R^-) = q_r(R^-) + k_e\lambda \left.\frac{\partial T(r,t)}{\partial r}\right|_{r=R^-}, \tag{11}$$

evaluating the influence of convection inside the droplet by effective conductivity parameter $\lambda_e = k_e\lambda$ [19]

$$k_e = 1.86 + 0.86 \tanh\left[2.245 \log\left(\frac{Pe}{30}\right)\right]. \tag{12}$$

The temperature field of an evaporating sphere in the case of unsteady radiative–conductive energy transfer is defined by the equation:

$$\frac{\partial T(r,t)}{\partial t} - a\frac{\partial^2 T(r,t)}{\partial r^2} - \frac{2a}{r}\frac{\partial T(r,t)}{\partial r} = \frac{1}{\rho c_p r^2}\frac{\partial}{\partial r}[r^2 q_r(r,t)]. \tag{13}$$

Radiation flux is calculated by the integration of spectral intensity over the wave number ω, azimuthal angle φ and angle θ between the opposite direction of the normal to the surface element and the incident beam:

$$q_r = \int_0^\infty \int_0^{2\pi} \int_0^\pi I_\omega(\theta,\varphi)\sin\theta\cos\theta\, d\theta\, d\varphi\, d\omega. \tag{14}$$

The change of spectral intensity along direction s is defined by [33]:

$$\frac{\partial I_\omega}{\partial s} = k_\omega[n_\omega^2 I_{\omega 0} - I_\omega], \tag{15}$$

If the initial

$$T(r) = T_0, \quad R = R_0, \quad I_\omega(R) = I_\omega(R_0), \quad \text{when} \quad t' = 0, \tag{16}$$

and boundary

$$T(R) = T_s(t), \quad R = R(t) \quad I_\omega(R) = I_\omega[R(t)], \quad \text{when } t' = t, \tag{17}$$

heat and mass transfer conditions are valid and according to the statement that the right hand side member of Eq. (13) is determined at that time, when the radiation flux density $q_r(r,t)$ is calculated according to a more precise temperature field in a droplet, achieved in the earlier iteration, the radiation–conduction Eqs. (13)–(17) problem is brought into the Dirichle problem [35], with the help of function $f(r,t) = r[T(r,t) - T_s(t)]$ [24]. The solution of the Dirichle problem is an integral temperature field equation:

$$T(r,t) = T_s(t) + \frac{2}{r}\sum_{n=1}^{\infty} \sin\frac{n\pi r}{R}\int_0^t f_n(R,t') \times \exp\left[-a\left(\frac{n\pi}{R}\right)^2(t-t')\right]dt', \tag{18}$$

where

$$f_n(R,t') = (-1)^n \frac{R}{n\pi}\frac{\mathrm{d}T_s}{\mathrm{d}t} + \frac{1}{R\rho c_p}\int_0^R q_r(r',t') \times \left(\sin\frac{n\pi r'}{R} - \frac{n\pi r'}{R}\cos\frac{n\pi r'}{R}\right)\mathrm{d}r'. \quad (19)$$

As the change of spectral intensity along direction s is defined by the change in the direction of a sphere radius (Fig. 1):

$$\frac{\mathrm{d}I_\omega}{\mathrm{d}s} = \pm\frac{(r^2 - R^2\sin^2\beta)^{1/2}}{r}\frac{\mathrm{d}I_\omega}{\mathrm{d}r}, \quad (20)$$

and Eq. (15) is multiplied by an integrating factor

$$\exp\left[\pm\int_{r_j}^{r}\frac{r'k_\omega\,\mathrm{d}r'}{\sqrt{r'^2 - R^2\sin^2\beta}}\right], \quad (21)$$

(where $r_j \equiv R$, sign '–' as $0 \le s < R\cos\beta$ and $r_j \equiv R\sin\beta$ sign '+' as $R\cos\beta \le s \le 2R\cos\beta$ [25]), Eq. (14) is transformed in the following way:

$$\begin{aligned}q_r(r) = 2\pi\int_0^\infty\int_0^{\pi/2} &\cos\gamma\sin\gamma[I_\omega(R,\gamma)\exp(-\tau_r^R) \\ &+ \int_r^R n_\omega^2 I_{0\omega}\exp(-\tau_r^{r'})\,\mathrm{d}\tau_r^{r'} \\ &- I_\omega(R,\gamma)\exp(-\tau_{r\sin\gamma}^R - \tau_{r\sin\gamma}^r) \\ &- \int_{r\sin\gamma}^r n_\omega^2 I_{0\omega}\exp(-\tau_{r'}^r)\,\mathrm{d}\tau_{r'}^r \\ &- \int_{r\sin\gamma}^R n_\omega^2 I_{0\omega}\exp(-\tau_{r\sin\gamma}^r - \tau_{r\sin\gamma}^{r'})\,\mathrm{d}\tau_r^{r'}]\,\mathrm{d}\gamma\,\mathrm{d}\omega. \end{aligned}\quad (22)$$

In Eq. (22) symbolic designations of the optical thickness and its derivative have the following meaning:

$$\tau_{r_i}^{r_j} = \int_{r_i}^{r_j}\mathrm{d}\tau_{r_i}^{r_j} = \int_{r_i}^{r_j}\frac{k_\omega\,\mathrm{d}r'}{\sqrt{1-(r/r')^2\sin^2\gamma}}, \quad (23)$$

where r_i and r_j correspond to the limits of optical thickness in Eq. (22). Angles θ, β, and γ have the following relation (Fig. 1):

$$R\sin\beta = r\sin\gamma, \quad \gamma = \pi - \theta. \quad (24)$$

$\gamma \equiv \beta$, when $r \equiv R$. The spectral intensity from the inner side of the droplet surface $I_\omega(R,\gamma)$ consists of the intensity of light that has been reflected from the inner side of the droplet surface by angle β and the intensity of light energy incoming the droplet from outside in the same direction:

$$I_\omega(R,\gamma) = \frac{n_\omega^2(R)[1 - r_\omega(\beta)]I_{\omega 0}(T_b) + r_\omega(\beta)\int_{R\sin\gamma}^R n_\omega^2(r')I_\omega(r')[\exp(-\tau_{R\sin\gamma}^R - \tau_{R\sin\gamma}^{r'}) + \exp(\tau_{r'}^R)]\mathrm{d}\tau_R^{r'}}{1 - r_\omega(\beta)\exp(-\tau_{R\sin\gamma}^R)}. \quad (25)$$

After differentiation of the Eq. (18) along coordinate r, the equation of temperature fields derivative $\partial T/\partial r$ is received:

$$\frac{\partial T(r,t)}{\partial r} = 2\sum_{n=1}^\infty\left(\frac{n\pi}{rR}\cos\frac{n\pi r}{R} - \frac{1}{r^2}\sin\frac{n\pi r}{R}\right)\int_0^t f_n(R,t') \times \exp\left[-a\left(\frac{n\pi}{R}\right)^2(t-t')\right]\mathrm{d}t'. \quad (26)$$

When condition $r \equiv R$ is valid:

$$\left.\frac{\partial T(r,t)}{\partial r}\right|_{r=R^-} = \frac{2\pi}{R^2}\sum_{n=1}^\infty n(-1)^n\int_0^t f_n(R,t') \times \exp\left[-a\left(\frac{n\pi}{R}\right)^2(t-t')\right]\mathrm{d}t'. \quad (27)$$

Energy expenditure for a phase change is proportional to the liquid vapor flux density on a droplet surface:

$$q_f(R^+) = Lm(R^+). \quad (28)$$

The liquid–vapor flux density during the evaporation is defined by [24]:

$$m(R^+) = \frac{D}{T_{vg}(R^+)}\frac{\mu_g}{RR_*}\left\{p_v(T_s) - p_{v,\infty} + \left(\frac{\mu_v}{\mu_g}\right) \times \left[p\ln\frac{p - p_{v,\infty}}{p - p_v(T_s)} - p_v(T_s) + p_{v,\infty}\right]\right\}, \quad (29)$$

which evaluates the influence of the Stefan flow on the mass transfer. The evaporation dynamics of a spherically symmetric droplet is defined by an equation:

$$\frac{\mathrm{d}R}{\mathrm{d}t} = -\frac{m(R^+)}{\rho}. \quad (30)$$

The solution of Eqs. (29) and (30) for initial radius droplet R_0 is:

$$R^2(t) = R_0^2 - 2\int_0^t \rho \frac{D}{T_{vg}(R^+)} \frac{\mu_g}{R_*} \times \left\{ p_v(T_s) - p_{v,\infty} + \left(\frac{\mu_v}{\mu_g}\right) \times \left[p \ln \frac{p - p_{v,\infty}}{p - p_v(T_s)} - p_v(T_s) + p_{v,\infty} \right] \right\} dt'. \quad (31)$$

Eqs. (2)–(31) describe heat and mass transfer of a semitransparent liquid droplet evaporating in the radiating media and allow us to investigate numerically the peculiarities of radiative–conductive heat transfer in the droplet.

3. Numerical solution

Eqs. (2)–(31), which describe heat and mass transfer of a semitransparent liquid droplet, evaporating in the radiating media, can be solved numerically, by the iteration method. Using this method, number J of a control droplet cross-section is selected freely; the position of the cross-section is defined by dimensionless sphere coordinate η_j ($\eta = 0$ when $j = 1$; $\eta_j = 1$ when $j = J$). Control time t is selected and number I of time coordinate change steps is provided ($t_i = 0$ when $i = 1$; $t_i = t$ when $i = I$). The numeric calculation scheme of expression (Eq. (2)) is formed under the statement that surface temperature of an evaporating droplet is known during time t_i. According to this scheme the calculation program has been made in which the iterative calculations are proceeded by the fastest descent method. For every time t_i, the iterative calculations are proceeded until a certain temperature of an evaporating droplet $T_{s,i}^k$ is determined, at which the members of expression (2) satisfy the following condition:

$$\left| 1 - \frac{\sum q_i^k(R^-)}{\sum q_i^k(R^+)} \right| 100\% \leq 0.1\%. \quad (32)$$

When forming the numerical calculation scheme of the temperature field in a droplet, the following premises are made: the temperature derivative of a droplet surface is constant during the time change interval $\Delta t_i = t_{i+1} - t_i$, the change of the function (Eq. (19)) integral is negligible and the change of the radiation flux in the interval of a sphere coordinate change is linear. Then the parameters of radiative–conductive energy transfer in a droplet at an instant of time I and in a droplet cross-section j are calculated in the following way:

$$Y_{j,I} = C_I + \sum_{n=1}^{\infty} f_{n,j} \sum_{i=1}^{I-1} \left[(-1)^n \frac{R_i}{n\pi} \frac{T_{s,i+1} - T_{s,i}}{t_{i+1} - t_i} + \frac{E_i}{\rho_i c_{p,i} R_i} \right] \times \frac{1}{a_i}\left(\frac{R_i}{n\pi}\right)^2 \left\{ \exp\left[a_i \left(\frac{n\pi}{R_i}\right)^2 (t_{i+1} - t_I) \right] - \exp\left[a_i \left(\frac{n\pi}{R_i}\right)^2 (t_i - t_I) \right] \right\}, \quad (33)$$

where

$$E_i = R_i \sum_{j=1}^{J-1} \left(q_{r,j} - \frac{q_{r,j+1} - q_{r,j}}{\eta_{j+1} - \eta_j} \right) \times \left\{ \frac{2}{n\pi} [\cos(n\pi\eta_j) - \cos(n\pi\eta_{j+1}) + \eta_j \sin(n\pi\eta) - \eta_{j+1} \sin(n\pi\eta_{j+1})] \right\} + R_i \sum_{j=1}^{J-1} \frac{q_{r,j+1} - q_{r,j}}{\eta_{j+1} - \eta_j} \left\{ \left(\frac{3}{n^2\pi^2} - \eta_{j+1}^2 \right) \sin(n\pi\eta_{j+1}) - \left(\frac{3}{n^2\pi^2} - \eta_j^2 \right) \sin(n\pi\eta_j) - [\eta_{j+1} \cos(n\pi\eta_{j+1}) - \eta_j \cos(n\pi\eta_j)] \frac{3}{n\pi} \right\}. \quad (34)$$

Parameters C_I and $f_{n,j}$ individualize expression (33) for calculation of various parameters of unsteady radiative–conductive heat transfer in an evaporating droplet, in particular: temperature field, $Y_{j,I} \equiv T(r, t)$, $C_I \equiv T_{s,I}$ and $f_{n,j} = 2\sin(n\pi\eta_j)/r_j$; temperature gradient on a droplet surface, $Y_{j,I} \equiv \partial T/\partial r|_{r=R}$, $C_I = 0$ and $f_{n,j} = 2\pi n(-1)^n/R_I^2$; temperature in a droplet center, $Y_{j,I} \equiv T(0, t)$, $C_I \equiv T_{s,I}$ and $f_{n,j} = 2\pi n/R_I$; local temperature gradient, $Y_{j,I} \equiv \partial T/\partial r$, $C_I = 0$ and $f_{n,j} = 2\pi n \cos(n\pi\eta_j)/(r_j R_I) - 2\sin(n\pi\eta_j)/r_j^2$.

The local radiation flux density in a droplet is calculated according to the temperature field that has been made more precise in earlier iteration. The integral of the wave number in expression (22) is calculated numerically using the rectangular method, the integral of angle γ using the Gauss method, the subintegral function expressed by [36]:

$$F(\omega,\gamma,r) = I_\omega(R)\Big[\exp(-\tau_r^R) - \exp(-\tau^R_{r\,\sin\gamma} - \tau^r_{r\,\sin\gamma})\Big] + \sum_{j=kk}^{J-1} n^2_{\omega,j} I_{0\omega,j}[\exp(-\tau_r^{r_j}) - \exp(-\tau_r^{r_j+1})] - n^2_{\omega,j0} I_{0\omega,j0}[\exp(-\tau^r_{r\,\sin\gamma}) - \exp(-\tau^r_{r\,\sin\gamma} - \tau^{r_{j0}}_{r\,\sin\gamma})] - \sum_{j=j0}^{J-1} n^2_{\omega,j} I_{0\omega,j}[\exp(-\tau^r_{r\,\sin\gamma} - \tau^{r_j}_{r\,\sin\gamma}) - \exp(-\tau^r_{r\,\sin\gamma} - \tau^{r_{j+1}}_{r\,\sin\gamma})] - n^2_{\omega,j0} I_{0\omega,j0}[\exp(-\tau^r_{r_{j0}}) - \exp(-\tau^r_{r\,\sin\gamma})] - \sum_{j=j0}^{kk-1} n^2_{\omega,j} I_{0\omega,j}[\exp(-\tau^r_{r_{j+1}}) - \exp(-\tau^r_{r_j})]. \tag{35}$$

In the most common case optical thickness in expression (35) is calculated:

$$\tau^{r_j}_{r\,\sin\gamma} = \tau^{r_{j0}}_{r\,\sin\gamma} + \tau^{r_j}_{r_{j0}} = k_{\omega,j0}\sqrt{r_{j0}^2 - r^2\sin^2\gamma} + \sum_{jj=j0}^{j-1} \times k_{\omega,jj}\left(\sqrt{r^2_{jj+1} - r^2\sin\gamma} - \sqrt{r^2_{jj} - r^2\sin^2\gamma}\right), \tag{36}$$

under the validity of condition that $r_{j0-1} < r\sin\gamma \le r_{j0}$ Spectral optical effects on the interphase contact surface (light refraction angle, surface reflection index, the Brewster angle) are calculated according to technique [37].

During every iteration the convective heating intensity of an evaporating sphere is being adjusted by solving the equation system

$$B_i = \frac{c_{pg,i}(T_g - T_{s,i})}{L_i}\left[1 + k_e\frac{q_{1,i}(R_i^-)}{q_{c,i}(R_i^+)}\right]$$

$$q_{c,i}(R^+) = \frac{\lambda_{vg,i}(T_g - T_{s,i})}{2R_{i-1}}(2 + 0.57Re_i^{1/2}\,Pr_i^{1/3})(1 + B_i)^{-0.7}, \tag{37}$$

where $q_{1,i}$ (R^-) during the evaporation process can change its sign: it is said to have negative value when $\partial T/\partial r|_{r=R} > 0$.

The dynamics of an evaporating droplet change is calculated in the following way:

$$R_I^2 = R_0^2 - 2\sum_{i=1}^{I-1}\frac{(t_{i+1} - t_i)D_i\mu_v}{\rho_i R_* T_{s,i}}\left\{p_v(T_{s,i}) - p_{v,\infty} + \left(\frac{\mu_v}{\mu_g}\right)\left[p\ln\frac{p - p_{v,\infty}}{p - p_v(T_{s,i})} - p_v(T_{s,i}) + p_{v,\infty}\right]\right\}. \tag{38}$$

Condition (32) is satisfied when the temperature of the droplet surface has been calculated with the accuracy of ± 0.01 K [38]. Therefore, it is necessary to evaluate 150 members in the infinite sum of expression (33) and to choose such duration of time change interval Δt_i, that the mean mass temperature of the heating

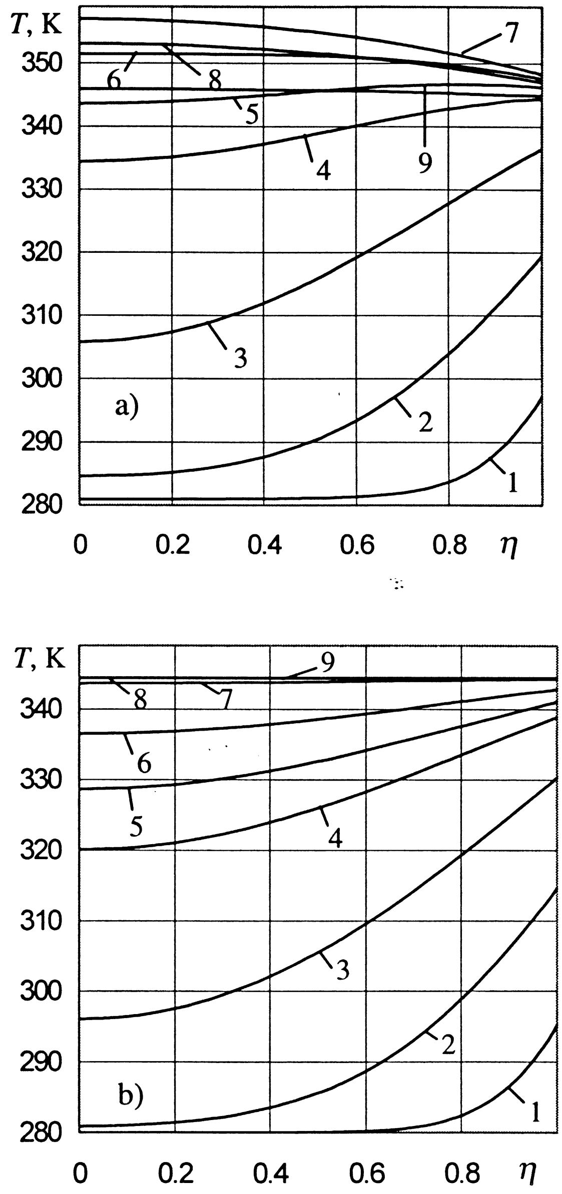

Fig. 2. Temperature field in an evaporating droplet in the case of radiative–convective (a) and convective (b) external heating (t, s: 1, 0.0012; 2, 0.0044; 3, 0.0102; 4, 0.0182; 5, 0.0226; 6, 0.0292; 7, 0.0481; 8, 0.108; 9, 0.1634; $T_g = 1500$ K; $R_0 = 100$ μm).

droplet change in it not more than by 1 K. In the calculation program the condition is satisfied automatically.

The accuracy of the radiation flux calculation depends on the chosen radiation spectrum interval and the number of its divisions. When choosing these divisions it is necessary to evaluate the peculiarities of the spectral optical characteristics of the semitransparent liquid. The spectral optical characteristics of water have been investigated very well [39,40]. The calculation results of the local radiation flux in water droplets stabilize due to expression (22) in the case when the radiation spectrum of $0.8 \div 800$ μm wavelength is divided into 150 parts [27]. The five-point Gauss scheme is used when expression (22) is integrated according to angle γ. The unsteady heat transfer has been examined numerically in evaporating water droplets of initial temperature 280 K. The droplets were carried by a radiating or non-radiating gas flow when $k_e = 1$; the temperature of the flow is T_d. The premise was made that the slip velocity of a droplet in gas equaled zero. The radiative–convective droplet heating was modeled on the condition that the radiation source is absolutely black and its temperature equals gas temperature.

4. Results and discussion

The change of droplet surface temperature, droplet lifetime, unsteady temperature field and total heat fluxes inside and outside the droplet were calculated numerically, as the droplet was evaporating in dry air at 373–1500 K temperature and under 0.1 MPa pressure. The heating dynamics of an evaporating droplet strictly depend on the method of droplet heating (Fig. 2). When the droplet is heated by convection or conduction the entire outer heating energy is supplied to the droplet surface. Some heat from hot gas will be required to overcome the latent heat of vaporization while some is transferred to the interior of the droplet. During the unsteady evaporation regime, the highest droplet temperature occurs on its surface. The droplet becomes isothermal (Fig. 2b) again at the moment of evaporation equilibrium setting.

A semitransparent liquid droplet absorbs radiation energy by its entire volume. Compared to the case of convection, the heating rate (Fig. 3) of the droplet increases significantly and the character of the temperature field gradient (Fig. 4) changes under the influence of absorbed radiation energy. According to the maximum place in the instant temperature field of the evaporating droplet it is possible to pick out three periods of the state change for an evaporating droplet: initial, during which the maximum temperature is on the droplet's surface; transient, during which the droplet's middle layers have a higher temperature, and final, during which the maximum temperature is in the droplet's center (Fig. 5). The time instant of equilibrium vaporization settling divides the final period into two parts: the first, during which the droplet is still being heated and the second, during which the droplet can even cool (Fig. 2a). Due to the radiation influence the droplet will reach the equilibrium vaporization regime in a non-isothermal state. At the moment of equilibrium vaporization beginning, the maximum local temperature field is reached inside the

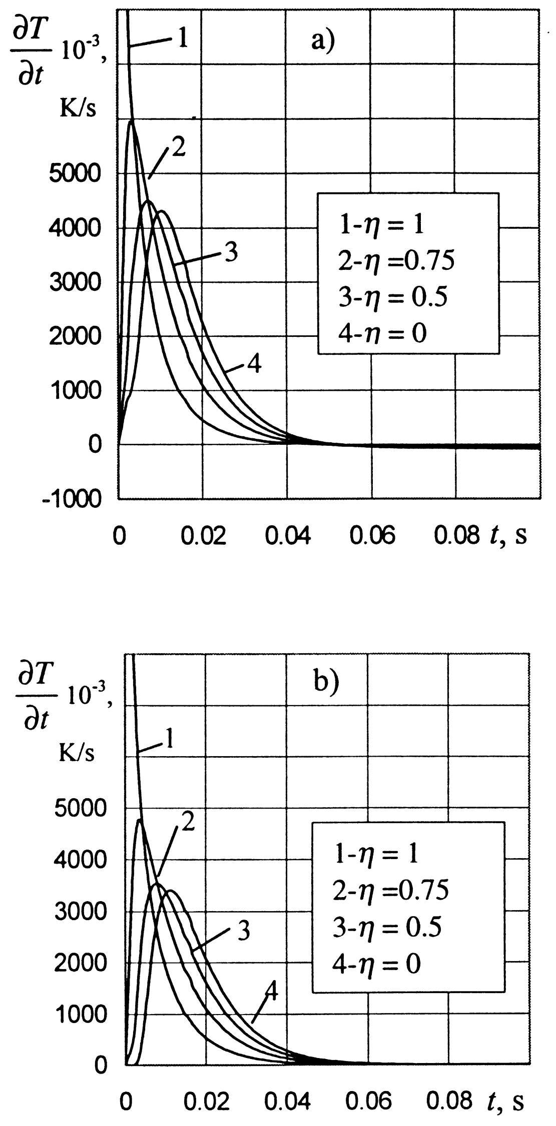

Fig. 3. Change of the heating rate of an evaporating droplet in the case of radiative–convective (a) and convective (b) external heating.

droplet (Fig. 2a, curve 7), and local temperature gradients ensure the withdrawal of absorbed energy into the droplet's surface. Then $q_l(r, t_p) = q_r(r, t_p)$, therefore $q_\Sigma(r, t_p) = 0$ (Fig. 6). The total heat flux represents the intensity of a droplet state change while the conduction component of the total heat flux describes the intensity of the energy distribution inside the droplet. The conductivity component vector during the initial period is directed to the center of the droplet and during the final period into the surface, while during the transient period the vector changes its direction in the cross-section at which local temperature gradient equals (Fig. 4a). Fig. 7 shows the peculiarities of the total heat flux radiation component change during the evaporation process. A negligible change in the radiation absorption character is caused by a change in the water optical characteristics, as a droplet is being heated intensively during the initial stage of the evaporation process. During the final evaporation stage radiation flux decreases, and this is because the change of density inside the droplet approaches a linear character (Fig. 7).

The magnitudes of the energy fluxes on the interphase contact surface are related by a complicated but strict interrelation of different physical nature transfer processes (Fig. 8). During the initial period the conductive heat flux on the inner droplet surface corresponds to that part of the external convective heat flux which heats the droplet. The magnitude of the flux decreases from $q_l(R^-, 0) \equiv q_c(R^+, 0)$ to zero (Fig. 8, curve 4). During the transient and the first part of the final periods the conductive heat flux on the inner droplet surface corresponds to that part of radiative heat flux which is taken into the droplet surface and is used for evaporation. During this time interval the above mentioned flux density increases from zero to $q_l(R^-, t_p) \equiv q_r(R^+, t_p)$. This value is reached at the beginning of equilibrium evaporation. The droplet is heated more under the influence of radiation (Fig. 2).

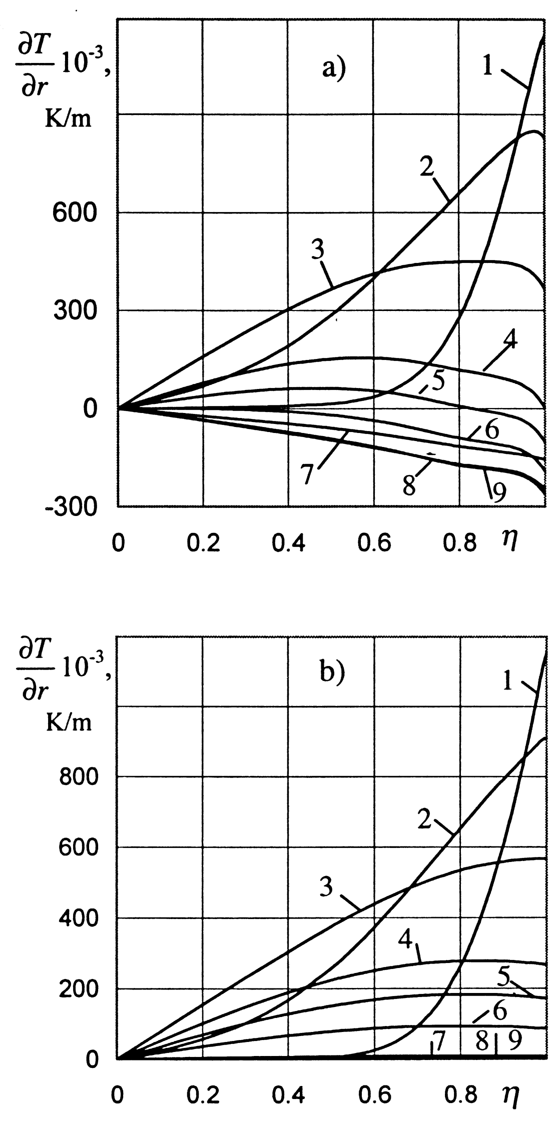

Fig. 4. Change of the temperature field gradient in an evaporating droplet (boundary conditions are the same as in Fig. 2).

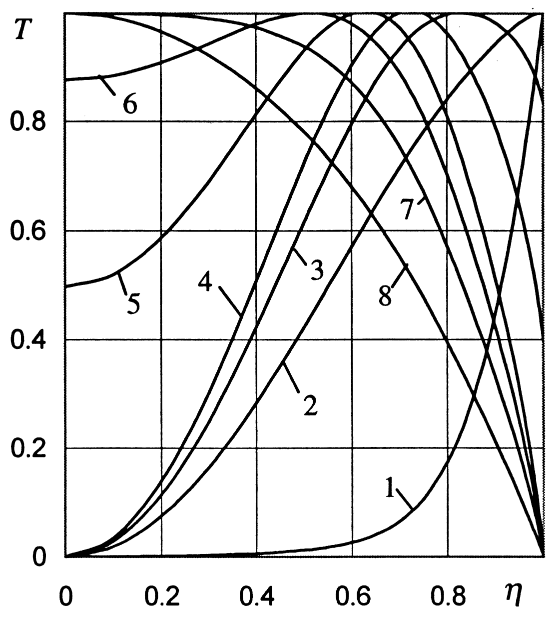

Fig. 5. Peculiarities of the temperature field in characteristic periods of the state change for evaporating droplets. $\bar{T} = [T(\eta, t) - T_{min}(t)]/[T_{max}(t) - T_{min}(t)]$. ($t$, s: 1, 0.0012; 2, 0.0182; 3, 0.0226; 4, 0.0241; 5, 0.0255; 6, 0.0270; 7, 0.0292; 8, 0.1634).

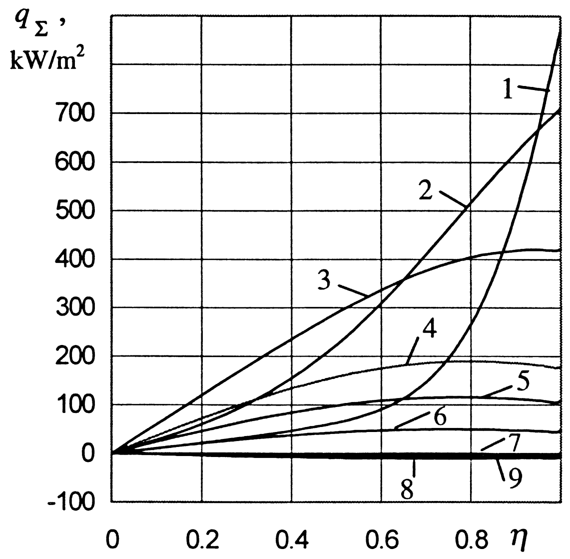

Fig. 6. Variation of the total heat flux in an evaporating droplet (t, s: 1, 0.0012; 2, 0.0044; 3, 0.0102; 4, 0.0182; 5, 0.0226; 6, 0.0292; 7, 0.0481; 8, 0.108).

The temperature increment of the evaporating droplet in the case of radiative–convective droplet heating compared to the droplet temperature settled down during the convective heating is proportional to the ratio between radiative and convective heat fluxes. A change in the droplet diameter acts on the above mentioned flux differently: when the intensity of the radiation flux density decreases the intensity of convective heating increases. During the second part of the final period the radiation intensity increases and the droplet diameter rapidly diminishes (Fig. 9). During this period the ratio of radiation and convection heat fluxes decreases, and the corresponding temperature increment changes while the droplet cools. Therefore, the magnitude

of the heat flux density of conduction on the inner droplet surface during this period of time depends on the radiating heating intensity and the rate of the mass mean temperature change of an evaporating droplet:

$$q_l(R^-, t) = q_r(R^+, t) - \frac{1}{3}\rho c_p R \frac{dT_m}{dt}. \tag{39}$$

However, the contribution of the second member on the right hand side of Eq. (39) is negligible. This is shown by curves 5 and 3, also 4 and 2 which almost coincide when $t/t_p > 1$ (Fig. 8). The inter-distribution of energies, used for droplet heating and evaporation, is shown by the ratio of curve ordinates 5 and 6 (Fig. 8). A sufficient distinct area under curve 6 confirms the importance of the evaluation of droplet heating (unsteadiness of transfer processes) when calculating the evaporation of dispersed liquid.

Under the influence of heat radiation the intensity of evaporation increases significantly, hence the droplet evaporation time shortens and the duration of

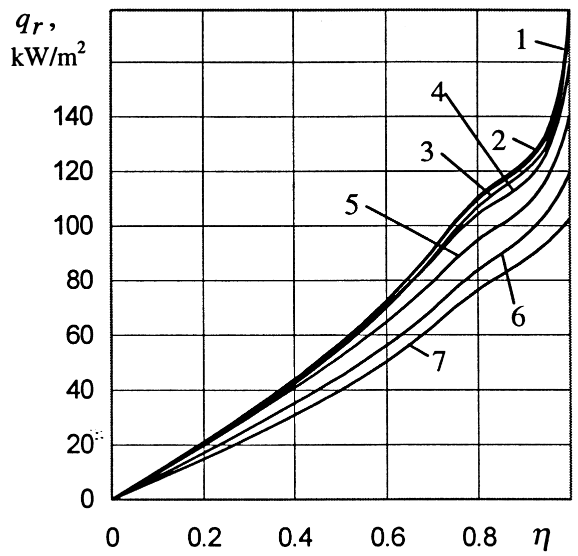

Fig. 7. Variation of the radiation heat flux in an evaporating droplet (t, s: 1, 0.0012; 2, 0.0292; 3, 0.0481; 4, 0.0108; 5, 0.1416; 6, 0.1576; 7, 0.1634).

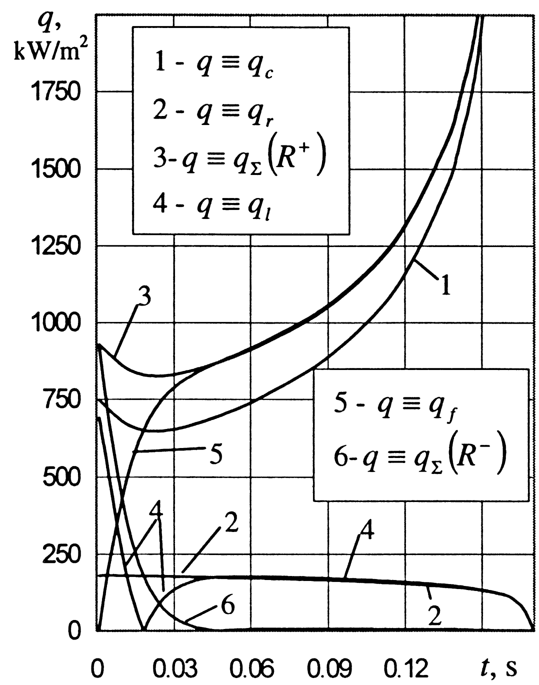

Fig. 8. Variation of the energy balance components on the surface of an evaporating droplet.

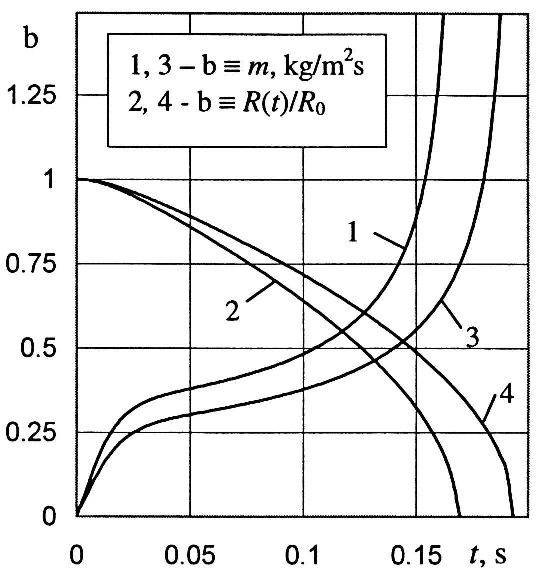

Fig. 9. Dependence of evaporation process on the heating way of droplet: 1 and 2, radiative–convective; 3 and 4, convective.

unsteady evaporation changes (Fig. 10). As the gas temperature increases and the droplet lifetime and duration of unsteady evaporation decreases their ratio also decreases (Fig. 10) and the change of heat flux ratios q_r/q_c and q_l/q_c is more distinct during the evaporation process (Fig. 11). At time instant $t/t_p = 1$ the above mentioned flux ratios are identical; later their magnitudes are close to each other. With convective heating the magnitude of number B changes only during unsteady evaporation (Fig. 12). Its change is stipulated by the changeable ratio between the energy that heats the droplet and the energy that causes phase changes. Due to the influence of radiation, heat transfer number B changes during the entire evaporation process. This is also stipulated by the change of the ratio between radiation and convection heat fluxes. This ratio approaches zero at the end of the evaporation process, therefore, the radiation influence is weakened. The calculated values of the heat transfer number correlate with the experimental results of other studies [14,16] (Fig. 12). The experiments have been carried out under the conditions of equilibrium evaporation of a water droplet in the presence of droplet convective and radiative–convective heating. During the numerical experiment slightly bigger values of number B in the case of convective heating have been received. This fact can be explained in the following way: when boundary conditions $q_r/q_c = 0$, $\Delta w = 0$ and $p_{v,\infty} = 0$ are

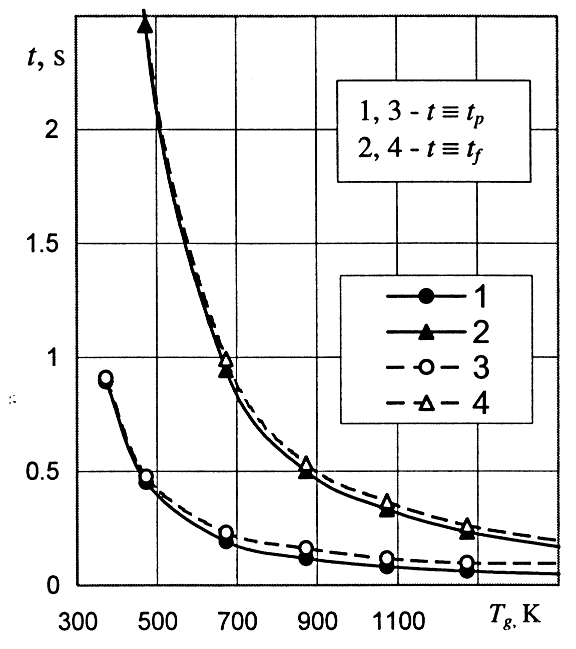

Fig. 10. Dependence of droplet lifetime and unsteady evaporation time on the gas temperature and the heating way of droplet: 1 and 2, radiative–convective; 3 and 4, convective ($R_0 = 100$ μm, $T_0 = 280$ K).

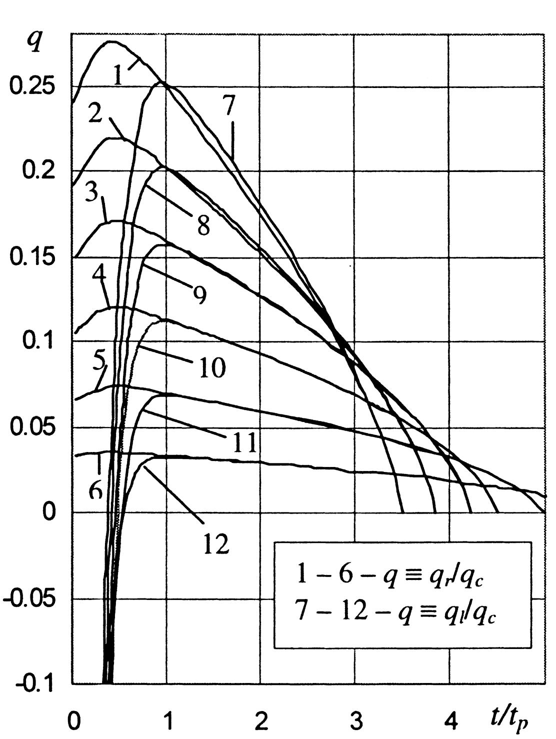

Fig. 11. Change of ratio between the components of total heat flux and the outer convective heat flux in an evaporating droplet (T_g, K: 1, 7–1500; 2, 8–1273; 3, 9–1073; 4, 10–873; 5, 11–673; 6, 12–473; $R_0 = 100$ μm).

valid, the droplet will be heated until the lowest possible temperature, which the droplet can acquire as it evaporates in a gas flow of constant temperature T_g. During the numerical study when radiative–convective heating was modelled, the condition $(q_r/q_c)_{max} < 0.173$, which was valid in study [16], remained active.

The above-suggested technique of a 'droplet' numerical study requires a lot of computer calculation time, because it is necessary to solve the problem of unsteady radiative–conductive energy transfer in a droplet many times. It is much easier to calculate conductive and radiative heat fluxes, not taking into account the interaction of energy transfer processes, so that the total heat flux q_Σ^0 is calculated by summing up heat fluxes q_l^0 and q_r^0. q_r^0 is calculated according to the temperature field which exists if energy is transferred in a droplet only by conduction. Due to the interaction of energy transfer processes $q_\Sigma \neq q_\Sigma^0$, but their ratio is proportional to the similarity parameter [30]:

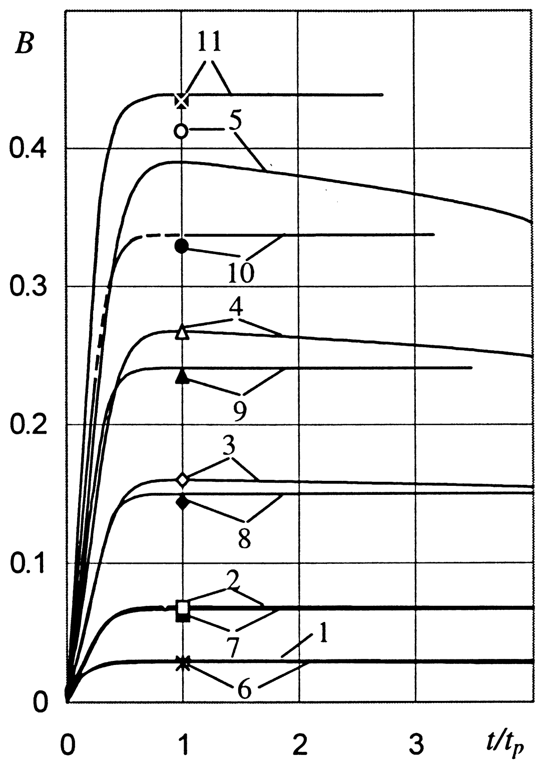

Fig. 12. Dependence of heat transfer number B on gas temperature and the heating way of the evaporating droplet: 1–5, radiative–convective; 6–11, convective. Lines, calculation results. Black dots, Yuen and Chen [14]; white dots, Renksizbulut and Yuen [16] experiment results (T_g, K: 1, 6–373; 2, 7–473; 3, 8–673; 4, 9–873; 5, 10–1073; 11–1273; $R_0 = 100$ μm; $q_r/q_k < 1.73$).

$$\frac{q_\Sigma}{q_\Sigma^0} = 1 + \frac{\partial \ln q_\Sigma^0}{\partial \ln \Delta h} \frac{\Delta h_r}{\Delta h}. \tag{40}$$

According to the statement that $\partial \ln q_\Sigma^0 / \partial \ln \Delta h \cong 1$ and that the ratio between changes in radiation and total process enthalpies is proportional to the ratio of heat fluxes q_r^0 and q_Σ^0 the similarity parameter in Eq. (4) acquires the expression used in study [29]:

$$\chi = \frac{q_r^0}{q_\Sigma^0}. \tag{41}$$

The results of the numerical study of radiative–convective heat transfer in an evaporating water droplet were compared when the interaction of energy transfer processes was taken into account and was neglected in calculations. The results of the experiment carried out showed the universality of the similarity parameter (Eq. (41)). The conductive heat transfer component of unsteady radiative–conductive heat transfer in evaporating droplets located in radiating media can be calculated by an empirical equation:

$$\frac{q_l}{q_\Sigma^0} = 1 - 0.022\chi - 10.9\chi^2 + 22\chi^3 - 19.6\chi^4 + 6.5\chi^5, \tag{42}$$

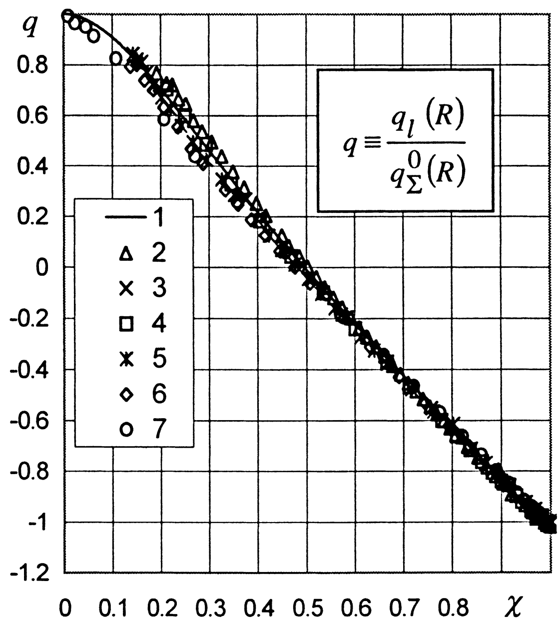

Fig. 13. Generalization of the study results for unsteady radiative–conductive heat transfer in an evaporating droplet: 1, results of calculation according to Eq. (42); 2–7, numerical experiment (T_g, K: 2, 1500; 3, 1273; 4, 1400; 5, 1073; 6, 873; 7, 373; R_0, μm: 2, 5, 6 and 7, 100; 3, 50, 4, 75).

which significantly simplifies the engineering calculation of unsteady transfer processes in radiating dispersed flows.

5. Conclusion

Complex heat and mass transfer processes inside and outside a droplet are closely related. The influence of heat radiation on the change of the state of evaporating semitransparent liquid droplets is important. While calculating the intensities of droplet external heating and evaporating, the temperature field and the total heat flux in it, it is necessary to take into account the unsteadiness of the transfer processes and their interaction. The evaporation intensity of semitransparent liquid droplets is defined by the total heat flux of external heating. The intensity of the unsteady droplet evaporation is defined by the difference between the heat flux of external heating and the total heat flux inside the droplet. The ratio between heat fluxes $q \equiv \lfloor q_{\Sigma}(R^-) - q_r(R^-) \rfloor / q_c(R^+)$ has a significant influence on the peculiarities of evaporating and heating droplet state changes. In the case of radiative–conductive droplet heating, $q \equiv q_l(R^-)/q_c(R^+)$. Under equilibrium droplet evaporation conditions q is close to the ratio between the heat flux of the external combined heat transfer components $q_r(R^+)/q_c(R^+)$. The combined unsteady heat transfer in evaporating semitransparent liquid droplets can be generalized using similarity theory methods.

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